Class X Session 2024-25
Subject - Mathematics (Basic)
Sample Question Paper - 12

Time: 3 Hours. **Total Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A - E.
2. Section A has 18 multiple choice questions and 2 Assertion-Reason based questions carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case study-based questions carrying 4 marks each with subparts of 1, 1, and 2 marks each, respectively.
7. All Questions are compulsory. However, an internal choice in 2 Question of Section B, 2 Questions of Section C and 2 Questions of Section D has been provided. An internal choice has been provided in all the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

Section A

Section A consists of 20 questions of 1 mark each.

Choose the correct answers to the questions from the given options. [20]

1. Given that HCF (306, 657) = 9, find LCM (306, 657).

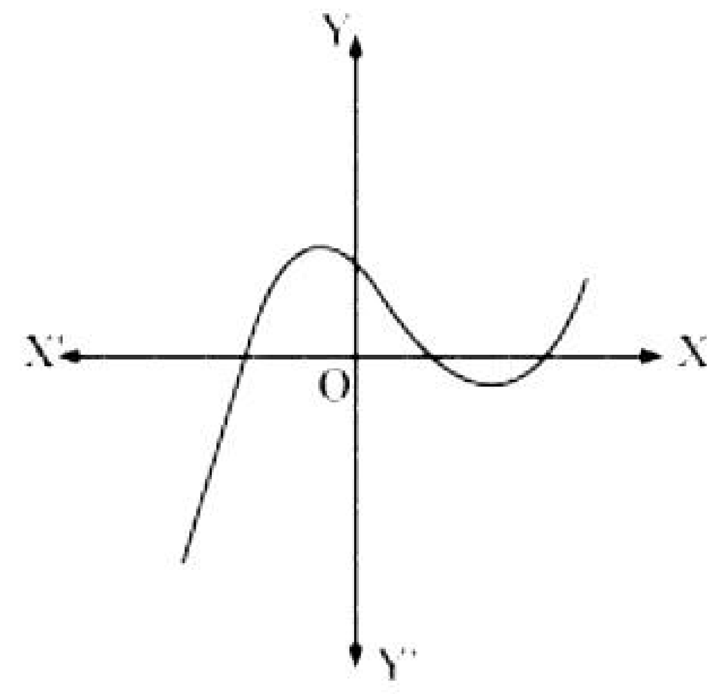
(a) 22338
(b) 22328
(c) 23238
(d) 23338

2. If p is a prime and p divides a^2, then p divides ____

(a) a - 2
(b) a + 2
(c) a/2
(d) a

3. Find the number of zeroes of p(x), in the following case.

(a) 1
(b) 2
(c) 3
(d) 4



4. Find the sum of zeroes of polynomial $3x^2 - x - 4 = 0$.
(a) 3
(b) 1/3
(c) -3
(d) -1/3

5. The quadratic polynomial with zeros 4 and 1 is ...
(a) $x^2 + 4x + 1$
(b) $x^2 - 4x - 1$
(c) $x^2 - 4x + 1$
(d) $x^2 + 4x - 1$

6. Find the distance between the points (2, 3) and (4, 1).
(a) $\sqrt{2}$
(b) $2\sqrt{2}$
(c) $3\sqrt{2}$
(d) $2\sqrt{3}$

7. All circles are __________
(a) equal
(b) congruent
(c) similar
(d) concentric

8. All ________ triangles are similar.
(a) scalene
(b) right angled
(c) isosceles
(d) equilateral

9. A tangent to a circle intersects it in ______ point(s).

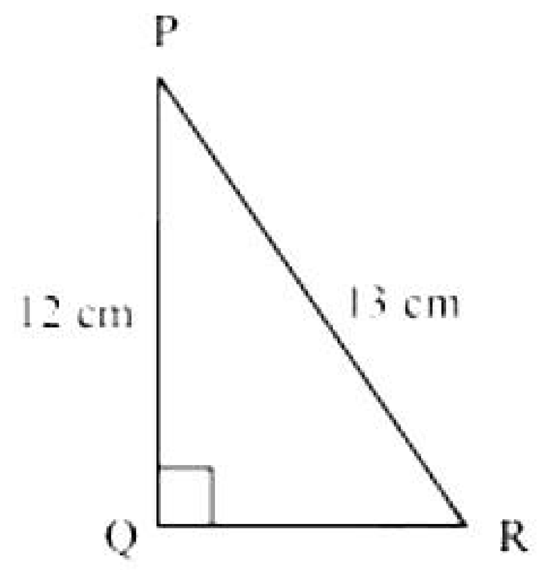
(a) one
(b) two
(c) three
(d) zero

10. In $\triangle ABC$ right angled at B, AB = 24 cm, BC = 7 m. Determine sin A.

(a) 7/24
(b) 7/25
(c) 24/25
(d) 24/7

11. In the given figure, find tan P.

(a) 13/12
(b) 12/13
(c) 5/13
(d) 5/12



12. Given, 15 sin A = 8. Find cosec A.

(a) 17/15
(b) 8/15
(c) 15/17
(d) 15/8

13. Find the area of a sector of a circle with radius 6 cm, if angle of the sector is 60°.

(a) 132/7 cm^2
(b) 132/9 cm^2
(c) 132/3 cm^2
(d) 132/17 cm^2

14. Find the area of a quadrant of a circle whose circumference is 22 cm.

(a) 77/6 cm^2
(b) 77/5 cm^2
(c) 77/9 cm^2
(d) 77/8 cm^2

15. Probability of an event E + Probability of the event 'not E'=________

(a) 0
(b) 1/2
(c) 1/4
(d) 1

16. The probability of an event that is certain to happen is 1, such an event is called_________.

(a) impossible event
(b) sure event
(c) equally likely event
(d) null event

17. If the perimeter and the area of a circle are numerically equal, then the radius of the circle is

(a) 2 units
(b) Π units
(c) 4 units
(d) 7 units

18. The sum of the probabilities of all elementary events of an experiment is ______.

(a) 0
(b) 1/2
(c) 1/4
(d) 1

DIRECTION: In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option.

19. Statement A (Assertion): Points (5, –2), (6, 4) and (7, –2) are the vertices of an isosceles triangle.

Statement R (Reason): Two sides of an isosceles triangle have equal length.

A. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
B. Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).
C. Assertion (A) is true but reason (R) is false.
D. Assertion (A) is false but reason (R) is true.

20. Statement A (Assertion): 6^n can end with the digit 0 for any natural number n.

Statement R (Reason): If any number ends with the digit 0, it should be divisible by 10.

A. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

B. Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

C. Assertion (A) is true but reason (R) is false.

D. Assertion (A) is false but reason (R) is true.

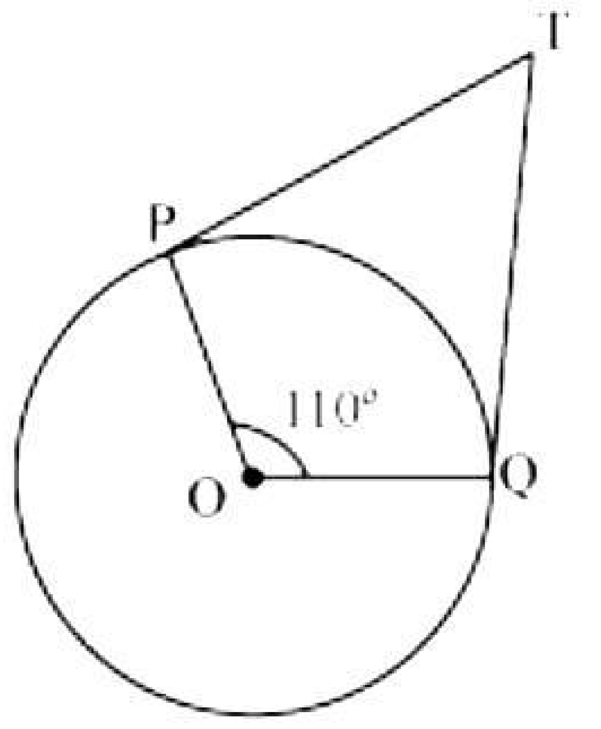
Section B

21. Find a quadratic polynomial with the sum and product of its zeroes as $\sqrt{2}$ and $\frac{1}{3}$ respectively. [2]

22. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. Find the radius of the circle. [2]

OR

In the given figure, if TP and TQ are the two tangents to a circle with centre O such that $\angle POQ = 110^\circ$, then find $\angle PTQ$.



23. Prove that the tangents drawn at the ends of a diameter of a circle are parallel. [2]

24. Evaluate: $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$ [2]

25. [2]

A car has two wipers which do not overlap. Each wiper has blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades. [2]

OR

To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use $\pi = 3.14$)

Section C

Section C consists of 6 questions of 3 marks each.

26. Prove that $\sqrt{5}$ is irrational. [3]

27. Which term of the A.P. 3, 8, 13, 18, ... is 78? [3]

28. Check whether –150 is a term of the A.P. 11, 8, 5, 2,... [3]

OR

Find the 31^{st} term of an A.P. whose 11^{th} term is 38 and the 16^{th} term is 73.

29. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle. [3]

30. Prove the following. [3]

$$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\operatorname{cosec}\theta$$

OR

$$\frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\operatorname{cosec}A+\cot A$$

31. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data: [3]

Number of cars	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
Frequency	7	14	13	12	20	11	15	8

Section D

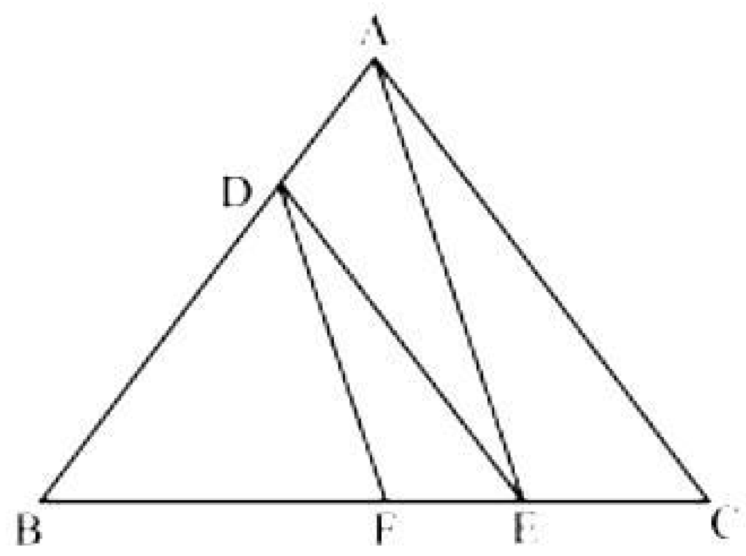
Section D consists of 4 questions of 5 marks each.

32. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without considering the time they stop at intermediate stations). If the average speed of the express train is 11 km/h more than that of the passenger train, find the average speeds of the two trains. [5]

OR

Two pipes running together can fill a cistern in $3\frac{1}{13}$ minutes. If one pipe takes 3 minutes more than the other to fill it, then find the time in which each pipe would fill the cistern.

33. In figure, DE || AC and DF || AE. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$ [5]



34. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm. ($\pi = 3.14$) [5]

OR

A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter with diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements. ($\pi = 3.14$).

35. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting [5]

(i) a king of red colour.
(ii) a face card.
(iii) a red face card.
(iv) the jack of hearts.
(v) a spade.

Section E

Case study-based questions are compulsory.

36. Rukhsar is celebrating her birthday. She invited her friends. She bought a packet of chocolates which contains 120 chocolates. She arranges the chocolates such that in the first row there are 3 chocolates, in second there are 5 chocolates, in third there are 7 chocolates and so on.

i. Find the total number of rows of chocolates. [1]

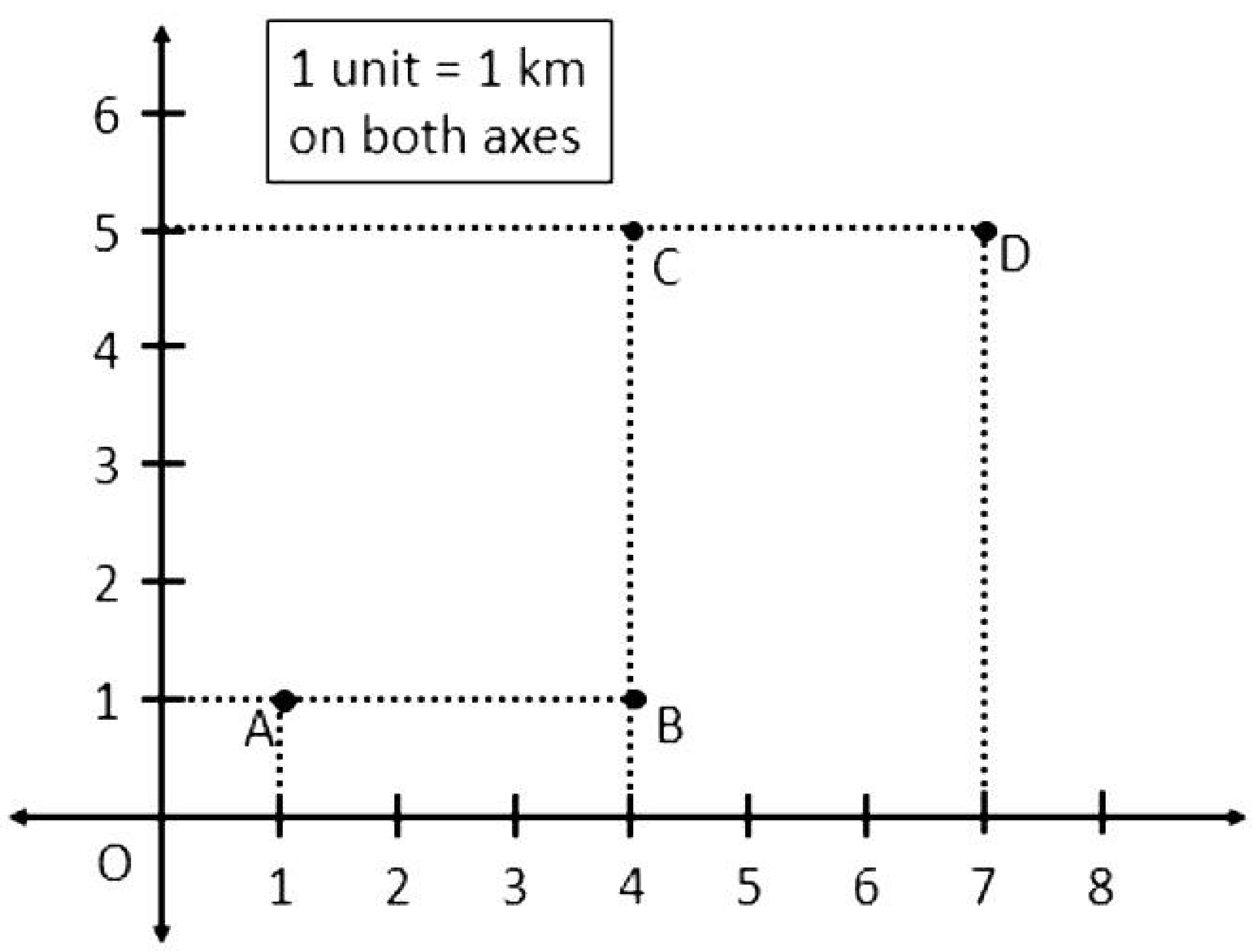
ii. How many chocolates are placed in last row? [2]

OR

Find the difference in number of chocolates placed in 7^{th} and 3^{rd} row. [2]

iii. If Rukhsar decides to make 15 rows, then how many total chocolates will be placed by her with the same arrangement? [1]

37. Amey runs a grocery store that offers home delivery of fresh groceries to its customers. His store is located at location A as indicated in the graph below. Now, he receives regular orders from the families living in the colonies located at B, C and D. Now, using the data given, answer the following questions.



i. Find the shortest distance between locations A and C. [2]

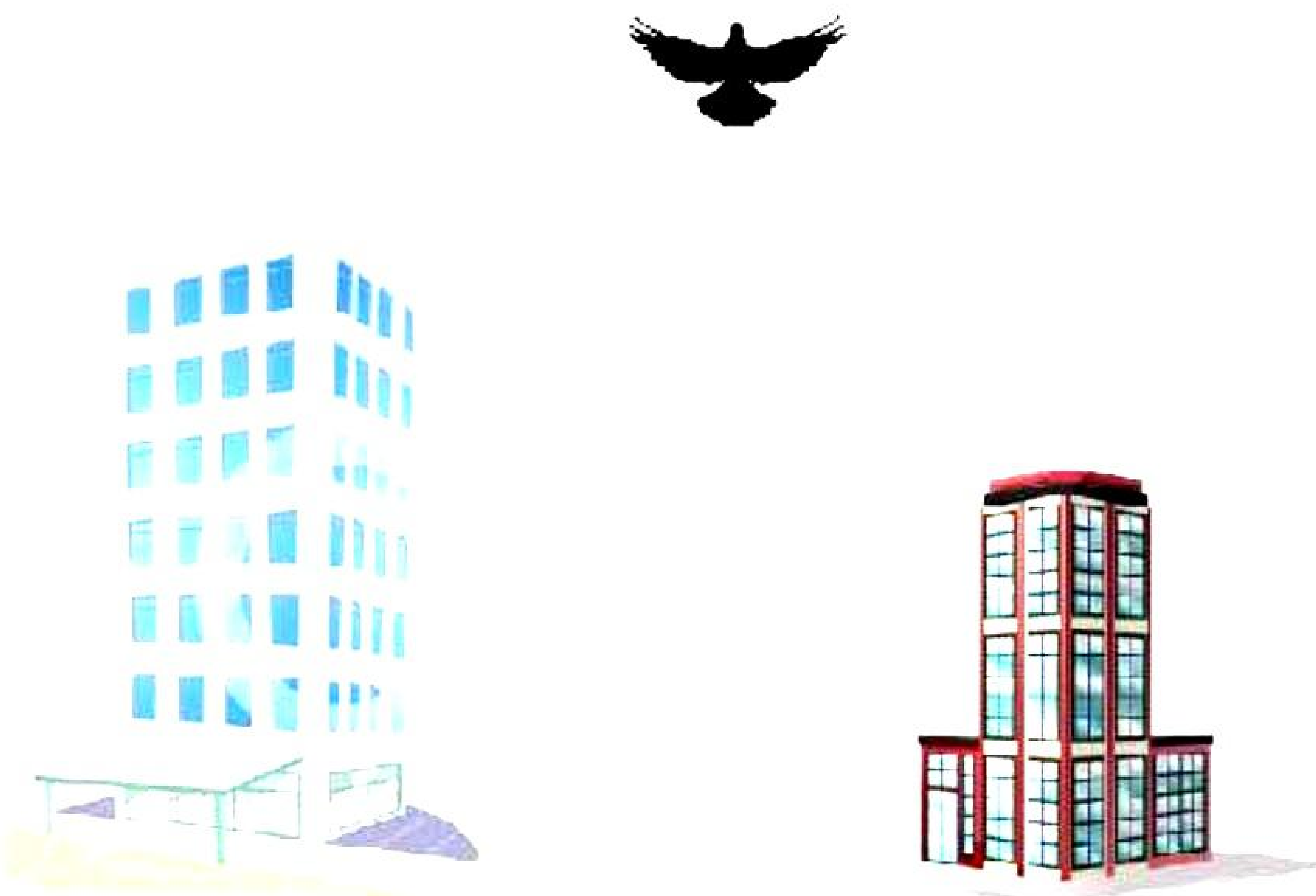
OR

Find the shortest distance between locations B and D. [2]

ii. Find the shortest distance between locations B and A. [1]

iii. Find the shortest distance between locations C and B. [1]

38. A bird flying at height h can see the top of the two buildings of height 534 m and 300 m. The angles of depression from bird to the top of first and second buildings are 30° and 60° respectively. If the distance between the two buildings is 142 m, and the bird is vertically above the midpoint of the distance between the two buildings, answer the following questions.



Building I Building II

i. Find the distance between bird and top of building I. [2]

OR

Find the distance between bird and top of building II. [2]

ii. Find the approximate height at which the bird is flying. [1]

iii. Find the angle of elevation if the bird is at the ground and its distance from building II is 300 m. [1]

Solution

Section A

1.

Correct option: (a)

Explanation:

$\text{HCF}(306, 657) = 9$

We know that, $\text{LCM} \times \text{HCF} = \text{Product of two numbers}$

$\therefore \text{LCM} \times \text{HCF} = 306 \times 657$

$$\text{LCM} = \frac{306 \times 657}{\text{HCF}} = \frac{306 \times 657}{9} = 22338$$

2.

Correct option: (d)

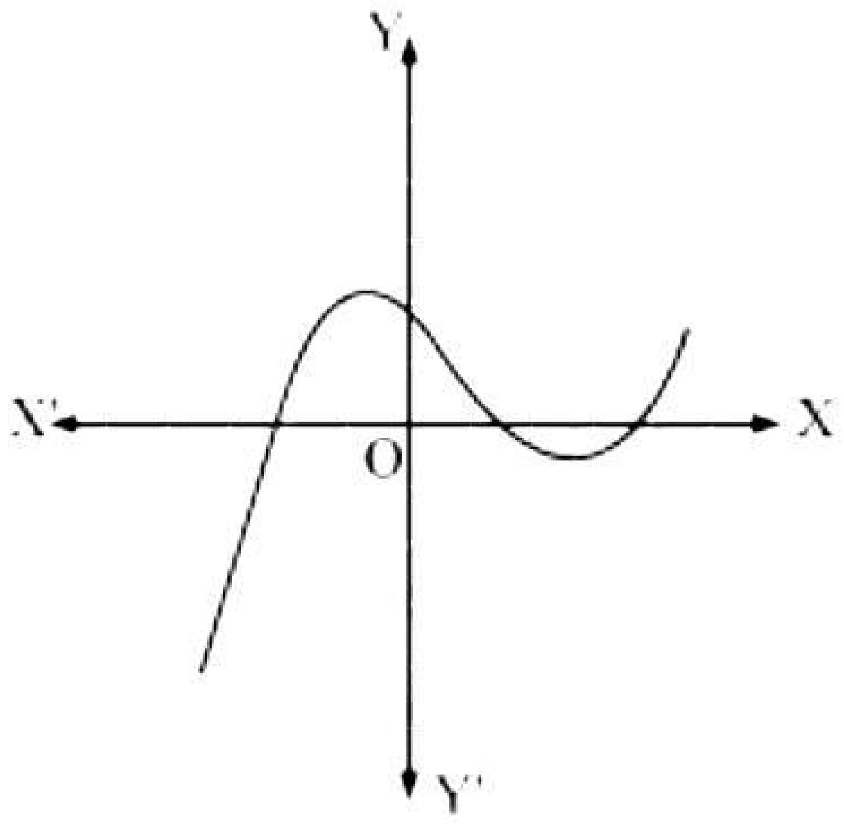
Explanation:

If p is a prime and p divides a^2, then p divides a, where a is a positive integer.

3.

Correct option: (c)

Explanation:



The graph cuts x-axis on 3 different points, hence it has 3 zeroes.

4.

Correct Option: (b)

Explanation:

$$\text{Sum of zeroes } = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2} = \frac{-(-1)}{3} = \frac{1}{3}$$

5.

Correct Option: (c)

Explanation:

Let the polynomial be $ax^2 + bx + c$.

$$\alpha + \beta = 4 = \frac{4}{1} = \frac{-b}{a}$$

$$\alpha \times \beta = 1 = \frac{1}{1} = \frac{c}{a}$$

If $a = k$, then $b = -4k$, $c = k$

Therefore, the quadratic polynomial is $k(x^2 - 4x + 1)$, where k is a real number.

6.

Correct Option: (b)

Explanation:

Distance between (2, 3) and (4, 1)

$$= \sqrt{(2-4)^2 + (3-1)^2}$$

$$= \sqrt{(-2)^2 + (2)^2}$$

$$= \sqrt{8}$$

$$= 2\sqrt{2}$$

7.

Correct option: (c)

Explanation:

All circles are similar.

8.

Correct option: (d)

Explanation:

All equilateral triangles are similar.

9.

Correct Option: (a)

Explanation:

A tangent to a circle intersects it in one point.

10.

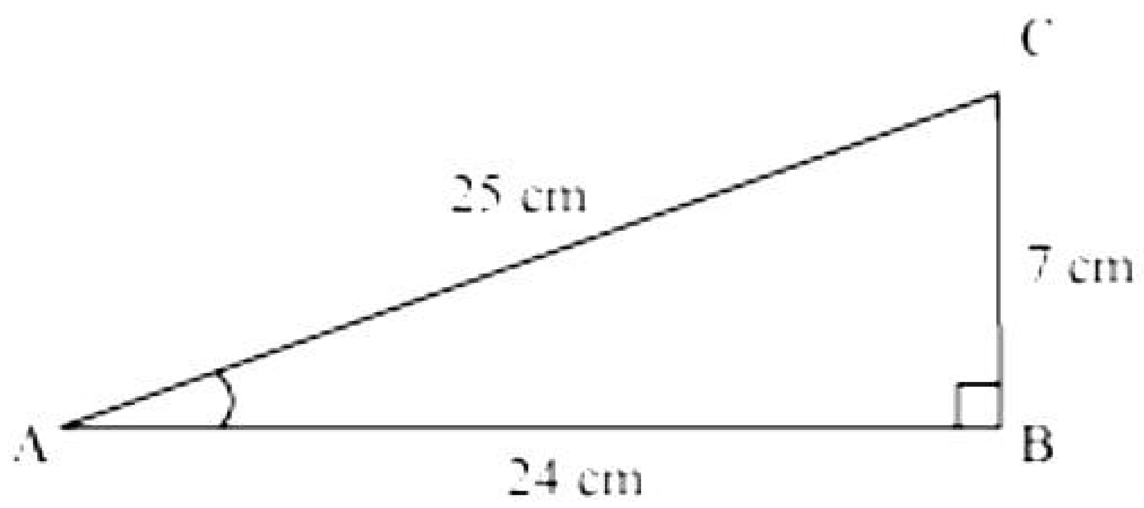
Correct Option: (b)

Explanation:

In ΔABC, by applying Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$= (24)^2 + (7)^2$
$= 576 + 49$
$= 625$
$AC = \sqrt{625} = 25$ cm

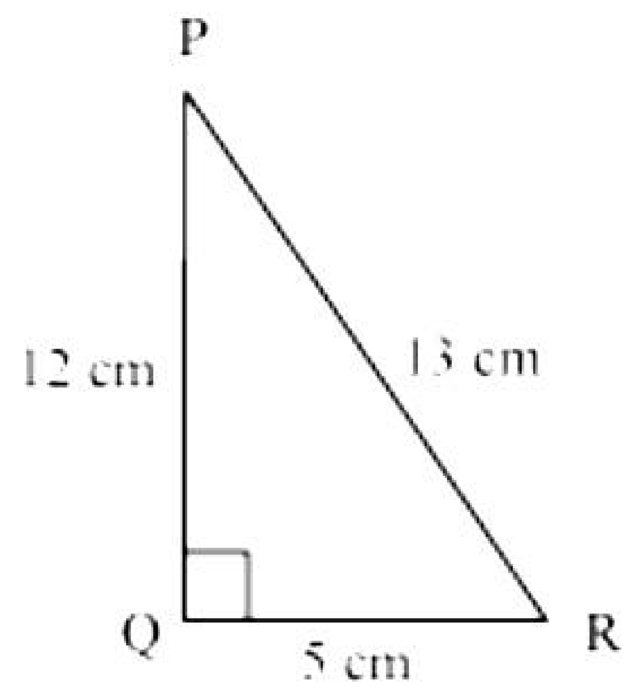


$$\sin A = \frac{\text{side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{7}{25}$$

11.

Correct option: (d)
Explanation:
In ΔPQR, by applying Pythagoras theorem,
$PR^2 = PQ^2 + QR^2$
$(13)^2 = (12)^2 + QR^2$
$169 = 144 + QR^2$
$25 = QR^2$
$QR = 5$

$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ} = \frac{5}{12}$$



12.

Correct Option: (d)
Explanation:
$15 \sin A = 8$

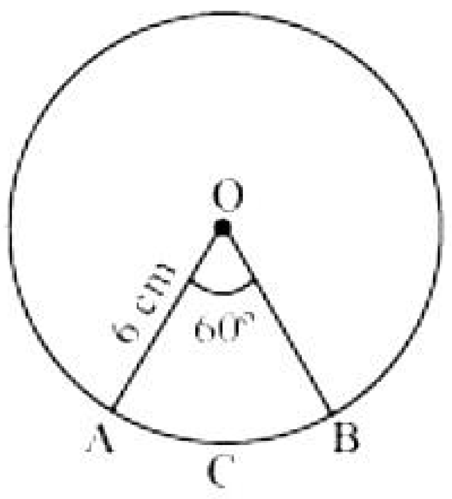
$$\Rightarrow \sin A = \frac{8}{15}$$

Also, $\operatorname{cosec} A = \frac{1}{\sin A} = \frac{15}{8}$

13.

Correct Option: (a)
Explanation:
Let OACB be a sector of circle making 60° angle at centre O of circle.

Area of sector OACB $= \frac{60°}{360°} \times \frac{22}{7} \times (6)^2 = \frac{1}{6} \times \frac{22}{7} \times 6 \times 6 = \frac{132}{7}$ cm^2



14.

Correct option: (d)

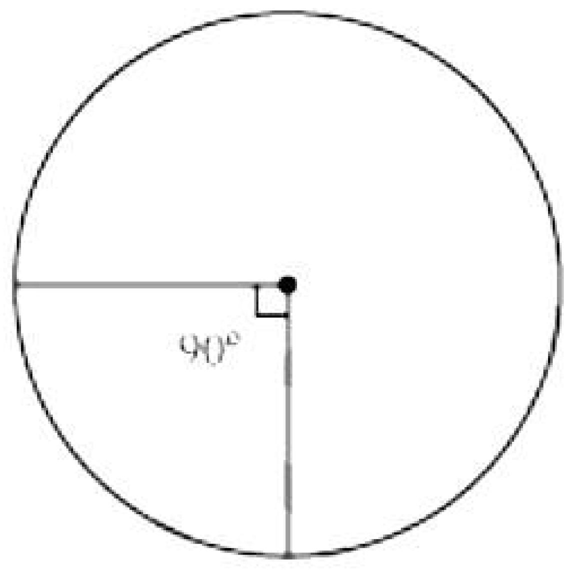
Explanation:

Let the radius of circle be r.

Circumference = 22 cm

$2\pi r = 22$

$r = \frac{22}{2\pi} = \frac{11}{\pi}$



Quadrant of a circle will subtend 90° angle at centre of circle.

So, area of such quadrant of circle $= \frac{90°}{360°} \times \pi \times r^2$

$= \frac{1}{4} \times \pi \times \left(\frac{11}{\pi}\right)^2$

$= \frac{121}{4\pi} = \frac{121 \times 7}{4 \times 22}$

$= \frac{77}{8}\ \text{cm}^2$

15.

Correct option: (d)

Explanation:

Probability of an event E + Probability of the event 'not E' = 1

16.

Correct Option: (b)

Explanation:

The probability of an event that is certain to happen is 1, such an event is called sure event.

17.

Correct option: (a)

Explanation:

Let the radius of circle be r.

Circumference of circle = $2\pi r$

Area of circle = πr^2

Given that circumference of circle and area of circle are equal.

So, $2\pi r = \pi r^2 \Rightarrow 2 = r$

So, the radius of the circle is 2 units.

18.

Correct Option: (d)

Explanation:

The sum of the probabilities of all elementary events of an experiment is **1**.

19.

Correct Option: (A)

Explanation:

Three non collinear points will represent the vertices of an isosceles triangle if its two sides are of equal length.

Hence, the reason statement is correct.

Let A = (5, –2), B = (6, 4), C = (7, –2)

$$AB=\sqrt{(5-6)^2+(-2-4)^2}=\sqrt{(-1)^2+(-6)^2}=\sqrt{1+36}=\sqrt{37}$$

$$BC=\sqrt{(6-7)^2+(4-(-2))^2}=\sqrt{(-1)^2+(6)^2}=\sqrt{1+36}=\sqrt{37}$$

$$CA=\sqrt{(5-7)^2+(-2-(-2))^2}=\sqrt{(-2)^2+0^2}=2$$

Here AB = BC

As two sides are equal in length therefore $\triangle ABC$ is an isosceles triangle.

Hence, both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

20.

Correct Option: (d)

Explanation:

If any number ends with the digit 0, it should be divisible by 10 or in other words its prime factorisation must include primes 2 and 5 both.

Hence, the reason statement is true.

Now,

Prime factorisation of $6^n = (2 \times 3)^n$

By Fundamental Theorem of Arithmetic, Prime factorisation of a number is unique.

So, 5 is not a prime factor of 6^n.

Hence, for any value of n, 6^n will not be divisible by 5.

Therefore, 6^n cannot end with the digit 0 for any natural number n.

Hence, the assertion statement is false.

Section B

21. Let the polynomial be ax^2+bx+c, and let its zeroes be α and β.

$\alpha+\beta=\sqrt{2}=\frac{3\sqrt{2}}{3}=\frac{-b}{a}$

$\alpha\beta=\frac{1}{3}=\frac{c}{a}$

If $a=3k$, then $b=-3k\sqrt{2}$, $c=k$

Therefore, the quadratic polynomial is $k(3x^2 - 3\sqrt{2}\,x + 1)$, where k is a real number.

22. Let O be the center of the circle.

OQ = 25 cm and PQ = 24 cm

Since, radius is perpendicular to tangent at the point of contact, $OP \perp PQ$.

Applying Pythagoras theorem in ΔOPQ,

$OP^2 + PQ^2 = OQ^2$

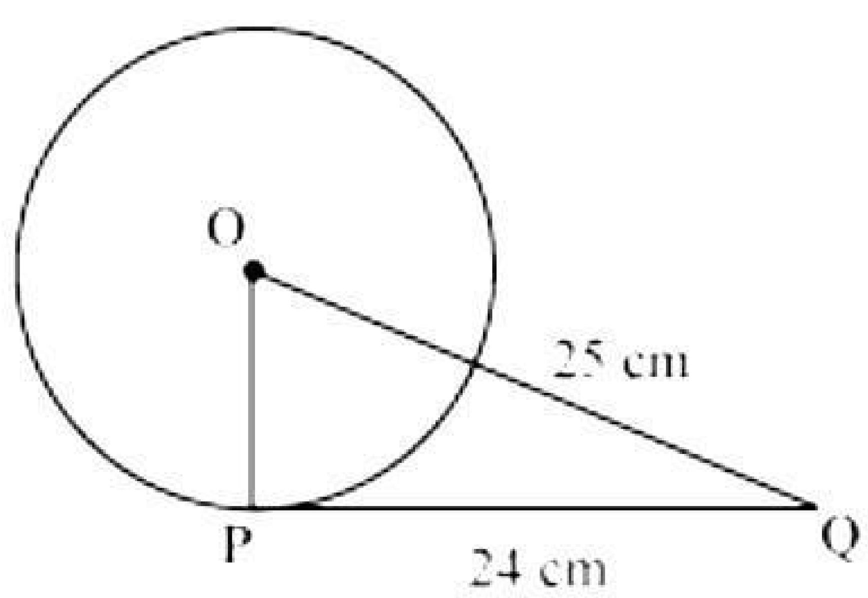
$\Rightarrow OP^2 + 24^2 = 25^2$

$\Rightarrow OP^2 = 625 - 576$

$\Rightarrow OP^2 = 49$

$\Rightarrow OP = 7$ cm

Thus, the radius of the circle is 7 cm.



OR

Since, radius is perpendicular to tangent at the point of contact.

$\therefore OP \perp TP$ and $OQ \perp TQ$.

$\Rightarrow \angle OPT = 90°$ and $\angle OQT = 90°$

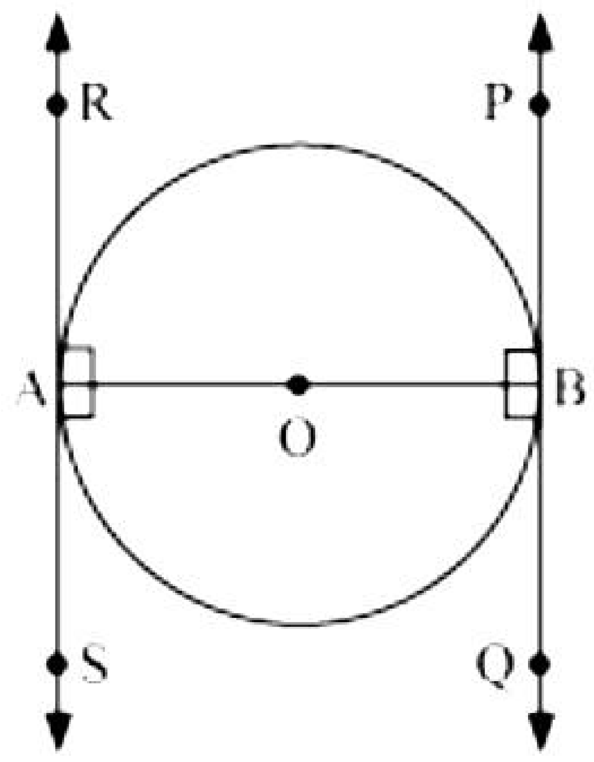
In the quadrilateral POQT,

$\angle OPT + \angle POQ + \angle OQT + \angle PTQ = 360°$

$90° + 110° + 90° + \angle PTQ = 360°$

$\therefore \angle PTQ = 70°$

23.



Let AB be a diameter of circle. Two tangents PQ and RS are drawn at the end points of the diameter AB.

It is known that the radius is perpendicular to tangent at the point of contact.

Therefore, $\angle OAR = 90°$, $\angle OAS = 90°$, $\angle OBP = 90°$ and $\angle OBQ = 90°$

$\Rightarrow \angle OAR = \angle OBQ$ ... (alternate interior angles)

$\Rightarrow \angle OAS = \angle OBP$ ... (alternate interior angles)

Since, alternate interior angles are equal, lines PQ and RS will be parallel.

24. $\sin 60° \cos 30° + \sin 30° \cos 60°$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$

25.

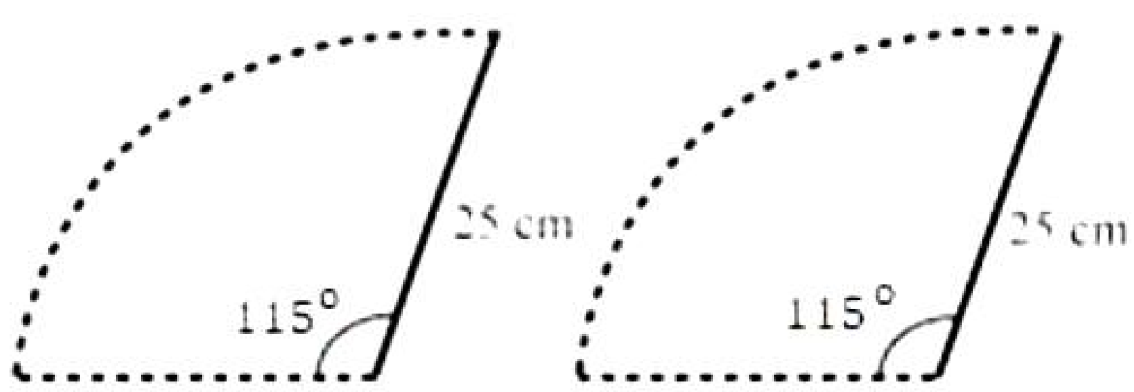


Figure shows that each blade of wiper will sweep an area of a sector of 115° in a circle of radius 25 cm.

Area of each sector $= \frac{115°}{360°} \times \pi r^2$

$$=\frac{23}{72}\times\frac{22}{7}\times25\times25$$

$$=\frac{158125}{252}\ \text{cm}^2$$

Area swept by 2 blades = $2\times\frac{158125}{252}$

$$=\frac{158125}{126}\ \text{cm}^2$$

OR

Lighthouse spreads light like a sector of angle 80° in a circle of radius 16.5 km.
Therefore, area of the sea over which the ships are warned = Area of sector

$$=\frac{80^\circ}{360^\circ}\times\pi r^2$$

$$=\frac{2}{9}\times3.14\times16.5\times16.5$$

$$=189.97\ \text{km}^2$$

Section C

26.

Let us assume, on the contrary that $\sqrt{5}$ is a rational number.

Therefore, we can find two integers a, b ($b \neq 0$) such that $\sqrt{5} = \frac{a}{b}$

Where a and b are co-prime integers.

$\sqrt{5} = \frac{a}{b}$

$\Rightarrow a = \sqrt{5}b$

$\Rightarrow a^2 = 5b^2$

Therefore, a^2 is divisible by 5 then a is also divisible by 5.

So, $a = 5k$, for some integer k.

Now, $a^2 = (5k)^2 = 5(5k^2) = 5b^2$

$\Rightarrow b^2 = 5k^2$

This means that b^2 is divisible by 5 and hence, b is divisible by 5.

This implies that a and b have 5 as a common factor.

And this is a contradiction to the fact that a and b are co-prime.

So, our assumption that $\sqrt{5}$ is rational is wrong.

Hence, $\sqrt{5}$ cannot be a rational number. Therefore, $\sqrt{5}$ is irrational.

27.

Given A.P. is 3, 8, 13, 18, ...

Here, $a = 3$ and $d = a_2 - a_1 = 8 - 3 = 5$

Let n^{th} term of this AP be 78.

$a_n = a + (n - 1)d$

$78 = 3 + (n - 1)5$

$(n - 1) = 15$

$n = 16$

Hence, 16^{th} term of this A.P. is 78.

28.

For this A.P., $a = 11$

$d = a_2 - a_1 = 8 - 11 = -3$

Let –150 be the n^{th} term of this AP.

$$a_n = a + (n-1)d$$
$$-150 = 11 + (n-1)(-3)$$
$$-161 = -3n + 3$$
$$-164 = -3n$$
$$n = \frac{164}{3}$$

Clearly, n is not an integer.

Therefore, –150 is not a term of this AP.

OR

Given that,

$a_{11} = 38$

$a_{16} = 73$

Now,

$a_n = a + (n - 1)d$

$a_{11} = a + (11 - 1)d$

$38 = a + 10d$ (1)

Similarly,

$a_{16} = a + (16 - 1)d$

$73 = a + 15d$ (2)

On subtracting (1) from (2), we obtain

$35 = 5d$

$d = 7$

From equation (1),

$38 = a + 10 \times 7$

$38 - 70 = a$

$a = -32$

$a_{31} = a + (31 - 1)d$

$= -32 + 30(7)$

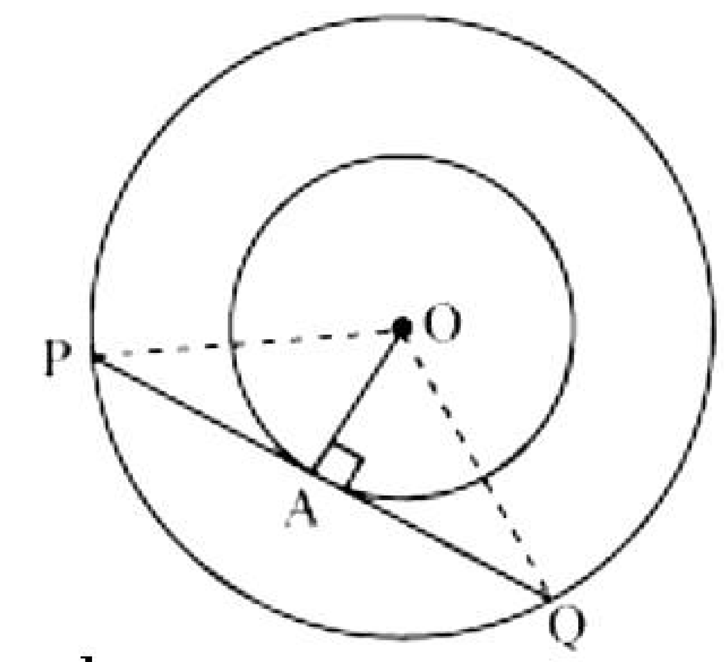
$= -32 + 210$

$= 178$

Hence, 31^{st} term of an A.P. is 178.

29.

Let two concentric circles be centered at point O. Let PQ be the chord of the larger circle which touches the smaller circle at point A. So, PQ is tangent to smaller circle.
Since, OA is the radius of circle, OA ⊥ PQ.
Applying Pythagoras theorem in ΔOAP,



$OA^2 + AP^2 = OP^2$

$3^2 + AP^2 = 5^2$

$AP^2 = 16$

AP = 4 cm

In ΔOPQ, as OA ⊥ PQ

⇒ AP = AQ (Perpendicular from the center of the circle bisects the chord)

∴ PQ = 2AP = 2 × 4 cm = 8 cm

So, the length of the chord of a larger circle is 8 cm.

30.

$$\text{L.H.S.} = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta}\right]$$

$$= \left(\frac{1}{\sin\theta-\cos\theta}\right)\left[\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta}\right]$$

$$= \left(\frac{1}{\sin\theta-\cos\theta}\right)\left[\frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta}\right]$$

$$= \frac{(1+\sin\theta\cos\theta)}{(\sin\theta\cos\theta)}$$

$$= \frac{1}{(\sin\theta\cos\theta)} + \frac{(\sin\theta\cos\theta)}{(\sin\theta\cos\theta)}$$

$$= \sec\theta\,\text{cosec}\,\theta + 1$$

$= \text{R.H.S.}$

OR

$$\text{L.H.S.} = \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$$

$$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}}$$

$$= \frac{\cot A - 1 + \text{cosec } A}{\cot A + 1 - \text{cosec } A}$$

$$= \frac{\{(\cot A) - (1 - \text{cosec } A)\}\{(\cot A) - (1 - \text{cosec } A)\}}{\{(\cot A) + (1 - \text{cosec } A)\}\{(\cot A) - (1 - \text{cosec } A)\}}$$

$$= \frac{(\cot A - 1 + \text{cosec } A)^2}{(\cot A)^2 - (1 - \text{cosec } A)^2}$$

$$= \frac{\cot^2 A + 1 + \text{cosec}^2 A - 2\cot A - 2\,\text{cosec} A + 2\cot A\,\text{cosec } A}{\cot^2 A - (1 + \text{cosec}^2 A - 2\text{cosec } A)}$$

$$= \frac{2\text{cosec}^2 A + 2\cot A\,\text{cosec } A - 2\cot A - 2\text{cosec } A}{\cot^2 A - 1 - \text{cosec}^2 A + 2\text{cosec } A}$$

$$= \frac{2\text{cosec } A(\text{cosec } A + \cot A) - 2(\cot A + \text{cosec } A)}{\cot^2 A - \text{cosec}^2 A - 1 + 2\text{cosec } A}$$

$$= \frac{(\text{cosec } A + \cot A)(2\text{cosec } A - 2)}{-1 - 1 + 2\text{cosec } A}$$ [Using the identity $\text{cosec}^2 A = 1 + \cot^2 A$]

$$= \frac{(\text{cosec } A + \cot A)(2\text{cosec } A - 2)}{(2\text{cosec } A - 2)}$$

$= \text{cosec } A + \cot A$

$= \text{R.H.S.}$

31.

From the given data we may observe that maximum class frequency is 20 belonging to class 40 – 50.

So, modal class = 40 – 50

Lower limit (l) of modal class = 40

Frequency (f_1) of modal class = 20

Frequency (f_0) of class preceding modal class = 12

Frequency (f_2) of class succeeding modal class = 11

Class size = 10

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 40 + \left[\frac{20 - 12}{2(20) - 12 - 11}\right] \times 10$$

$$= 40 + \left(\frac{80}{40 - 23}\right)$$

$$= 40 + \frac{80}{17}$$

$= 40 + 4.7$

$= 44.7$

So, the mode of this data is 44.7.

Section D

32. Let the average speed of a passenger train be x km/h.

$\Rightarrow$ Average speed of express train = (x + 11) km/h

It is given that the time taken by the express train to cover 132 km is 1 hour less than the passenger train to cover the same distance.

$$\therefore \frac{132}{x}-\frac{132}{x+11}=1$$

$$\Rightarrow 132\left[\frac{x+11-x}{x(x+11)}\right]=1$$

$$\Rightarrow \frac{132\times 11}{x(x+11)}=1$$

$$\Rightarrow 132\times 11=x(x+11)$$

$$\Rightarrow x^2+11x-1452=0$$

$$\Rightarrow x^2+44x-33x-1452=0$$

$$\Rightarrow x(x+44)-33(x+44)=0$$

$$\Rightarrow (x+44)(x-33)=0$$

$$\Rightarrow x=-44 \text{ or } x=33$$

Speed cannot be negative.

$\Rightarrow$ x = 33 and (x + 11) = 44

Therefore, the speed of the passenger train will be 33 km/h and that of the express train will be 44 km/h.

OR

Let the faster pipe take x minutes to fill the cistern.

Then the other pipe takes (x + 3) minutes.

$$\frac{1}{x}+\frac{1}{(x+3)}=\frac{13}{40}$$

$$\Rightarrow \frac{(x+3)+x}{x(x+3)}=\frac{13}{40}$$

$$\Rightarrow 40(2x+3)=13(x^2+3x)$$

$$\Rightarrow 80x+120=13x^2+39x$$

$$\Rightarrow 13x^2-41x-120=0$$

$$\Rightarrow 13x^2-65x+24x-120=0$$

$$\Rightarrow 13x(x-5)+24(x-5)=0$$

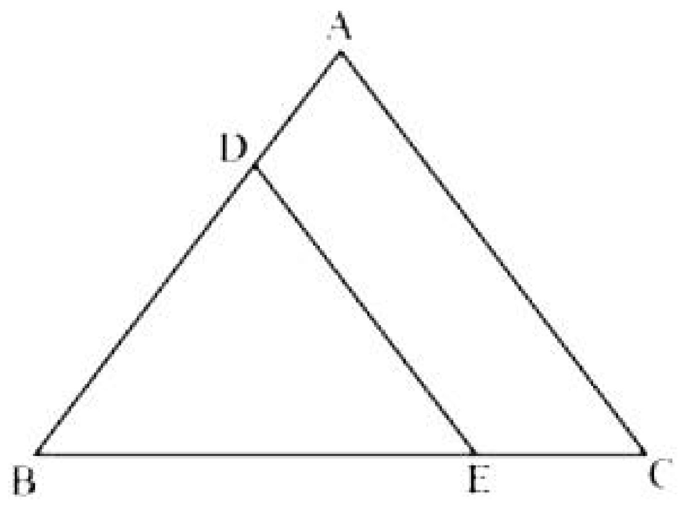
$$\Rightarrow (x-5)(13x+24)=0$$

$$\Rightarrow x=5 \quad \text{or} \quad x=\frac{-24}{13}$$

$$\Rightarrow x=5 \quad (\text{Time cannot be negative})$$

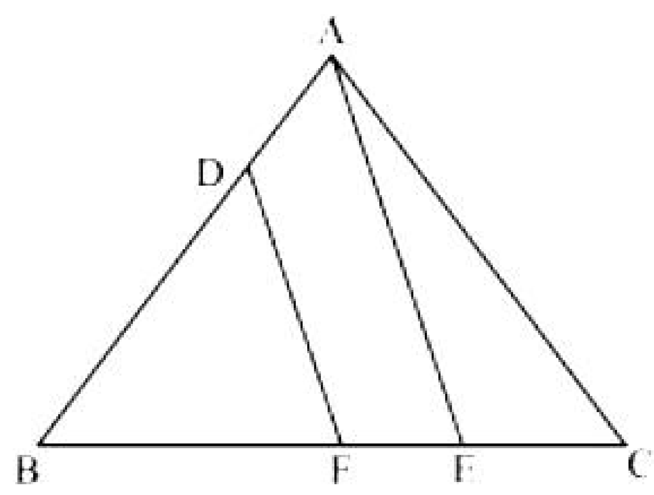
If the faster pipe takes 5 minutes to fill the cistern, then the other pipe takes (5 + 3) minutes = 8 minutes

33.



In ΔABC, $DE \parallel AC$.

$\therefore \frac{BD}{DA} = \frac{BE}{EC}$ ....(i)



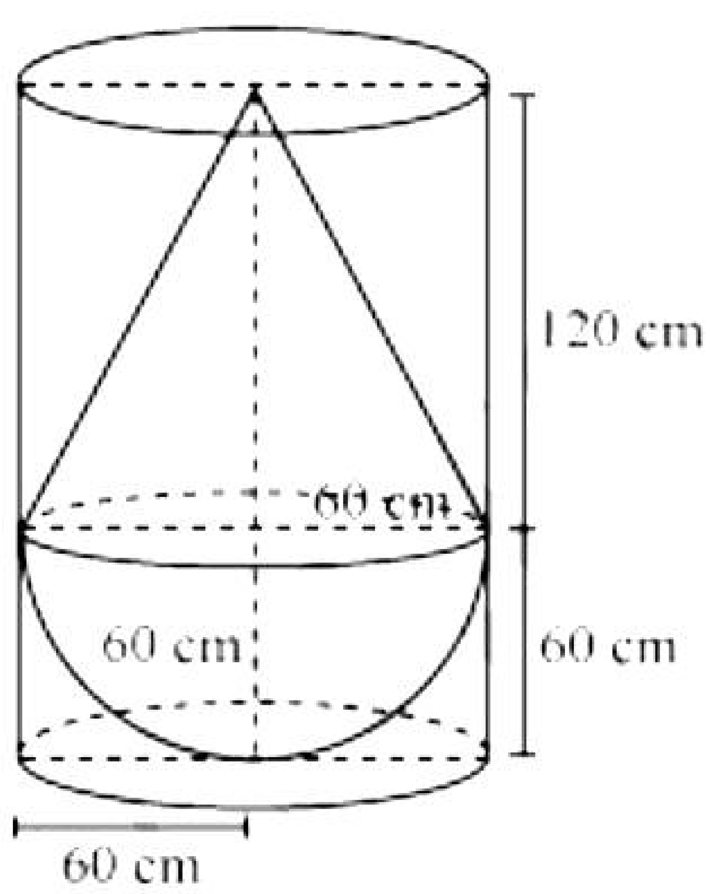
In ΔBAE, $DF \parallel AE$.

$\therefore \frac{BD}{DA} = \frac{BF}{FE}$ ....(ii)

From (i) and (ii),

$\frac{BE}{EC} = \frac{BF}{FE}$

34.



Radius (r) of hemispherical part = radius (r) of conical part = 60 cm

Height (h_2) of conical part of solid = 120 cm

Height (h_1) of cylinder = 120 + 60 = 180 cm

Radius (r) of cylinder = 60 cm

Volume of water left = volume of cylinder – volume of solid

$= \text{volume of cylinder} - (\text{volume of cone} + \text{volume of hemisphere})$

$= \pi r^2 h_1 - \left(\frac{1}{3}\pi r^2 h_2 + \frac{2}{3}\pi r^3\right)$

$= \pi(60)^2(180) - \left(\frac{1}{3}\pi(60)^2 \times 120 + \frac{2}{3}\pi(60)^3\right)$

$= \pi(60)^2\left[(180) - (40 + 40)\right]$

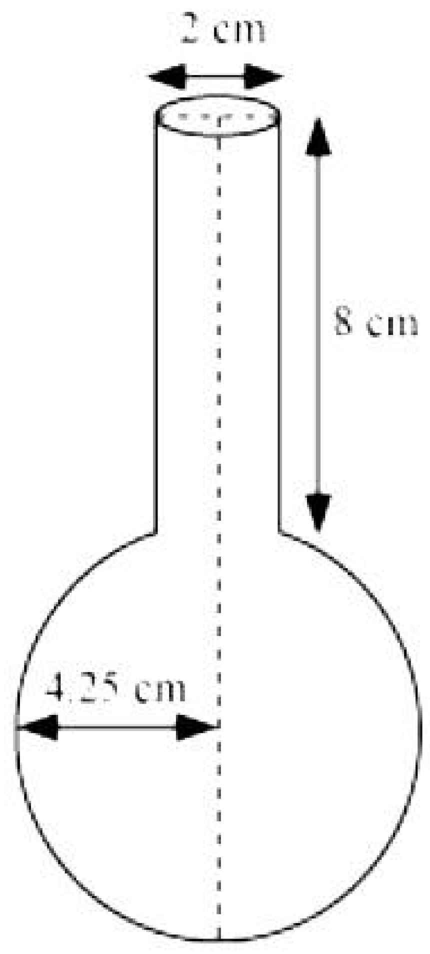
$= \pi(3600)(100)$

$= 360000\pi \text{ cm}^3$

$= 1130400 \text{ cm}^3$

$= 1.13 \text{ m}^3$

OR



Radius (r_1) of spherical part = 8.5/2 cm
Height (h) of cylindrical part = 8 cm

Radius (r_2) of cylindrical part = $\frac{2}{2} = 1$ cm

Volume of vessel = volume of sphere + volume of cylinder

$= \frac{4}{3}\pi r_1^3 + \pi r_2^2 h$

$= \frac{4}{3}\pi\left(\frac{8.5}{2}\right)^3 + \pi(1)^2(8)$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{8.5}{2} \times \frac{8.5}{2} \times \frac{8.5}{2} + \frac{22}{7} \times 8$

$= 321.39 + 25.12$

$= 346.51 \text{ cm}^3$

Hence, she is wrong.

35. Total number of possible outcomes = 52

(i)

Let A be the event that the card is a red king card.

There are 2 kings of red colour .

$\Rightarrow$ Number of favorable outcomes $= 2$

Therefore, $P(A) = \frac{2}{52} = \frac{1}{26}$

(ii)

Let B be the event that the card is a face card.

There are 12 face cards.

$\Rightarrow$ Number of favorable outcomes $= 4$

Therefore, $P(B)= \frac{12}{52} = \frac{3}{13}$

(iii)

Let C be the event that the card is a red face card.

There are 6 red face cards.

$\Rightarrow$ Number of favorable outcomes $= 6$

Therefore, $P(C) = \frac{6}{52} = \frac{3}{26}$

(iv)

Let D be the event that the card is a jack of heart.

There is 1 jack of heart.

$\Rightarrow$ Number of favorable outcomes $= 1$

Therefore, $P(D)= \frac{1}{52}$

(v)

Let E be the event that the card is a spade card.

There are 13 spade cards.

$\Rightarrow$ Number of favorable outcomes $= 13$

Therefore, $P(E) = \frac{13}{52} = \frac{1}{4}$

Section E

36.

i.

Here, the chocolates are arranged in increasing order of 2.
Thus, it forms an A.P. with $a = 3$ and $d = 2$.
Therefore, the required A.P. is 3, 5, 7, ...
Given, $S_n = 120$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\Rightarrow 120 = \frac{n}{2}\left[2\times 3 + (n-1)2\right]$$

$$\Rightarrow 240 = \left(6n + 2n^2 - 2n\right)$$

$$\Rightarrow n^2 + 2n - 120 = 0$$

$$\Rightarrow (n+12)(n-10) = 0$$

$$\Rightarrow (n+12) = 0 \text{ or } (n-10) = 0$$

$$\Rightarrow n = -12 \text{ or } n = 10$$

Number of rows can't be negative.

Hence, total number of rows of chocolates is 10.
Therefore the required A.P is 3, 5, 7, ...
$S_n = 120$ ..... given

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\Rightarrow 120 = \frac{n}{2}\left[2\times 3 + (n-1)2\right]$$

$$\Rightarrow 240 = \left(6n + 2n^2 - 2n\right)$$

$$\Rightarrow n^2 + 2n - 120 = 0$$

$$\Rightarrow (n+12)(n-10) = 0$$

$$\Rightarrow (n+12) = 0 \text{ or } (n-10) = 0$$

$$\Rightarrow n = -12 \text{ or } n = 10$$

Hence, the number of rows of chocolates will be 10.

ii.

Here, $a = 3$, $d = 2$ and $n = 10$

$a_n = a_{10} = a + (n - 1)d = 3 + (10 - 1)2 = 21$

Hence, 21 chocolates are placed in the last row.

OR

We have, $d = 2$ and $a_n = a + (n - 1)d$

$\Rightarrow a_7 - a_3 = a + 6d - a - 2d = 4d = 4(2) = 8$

Hence, the difference in number of chocolates placed in 7^{th} and 3^{rd} row is 8.

iii.

Here, n = 15

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\Rightarrow S_{15} = \frac{15}{2}\left[2\times3 + 14\times2\right] = \frac{15\times34}{2} = 255$$

Hence, 255 chocolates will be placed by her with the same arrangement.

37.

i.

$A(1,1)$ and $C(4,5)$

$$d(AC) = \sqrt{(4-1)^2 + (5-1)^2} = 5 \text{ km}$$

OR

$B(4,1)$ and $D(7,5)$

$$d(BD) = \sqrt{(7-4)^2 + (5-1)^2} = 5 \text{ km}$$

ii.

$B(4,1)$ and $A(1,1)$

$$d(BA) = \sqrt{(1-4)^2 + (1-1)^2} = 3 \text{ km}$$

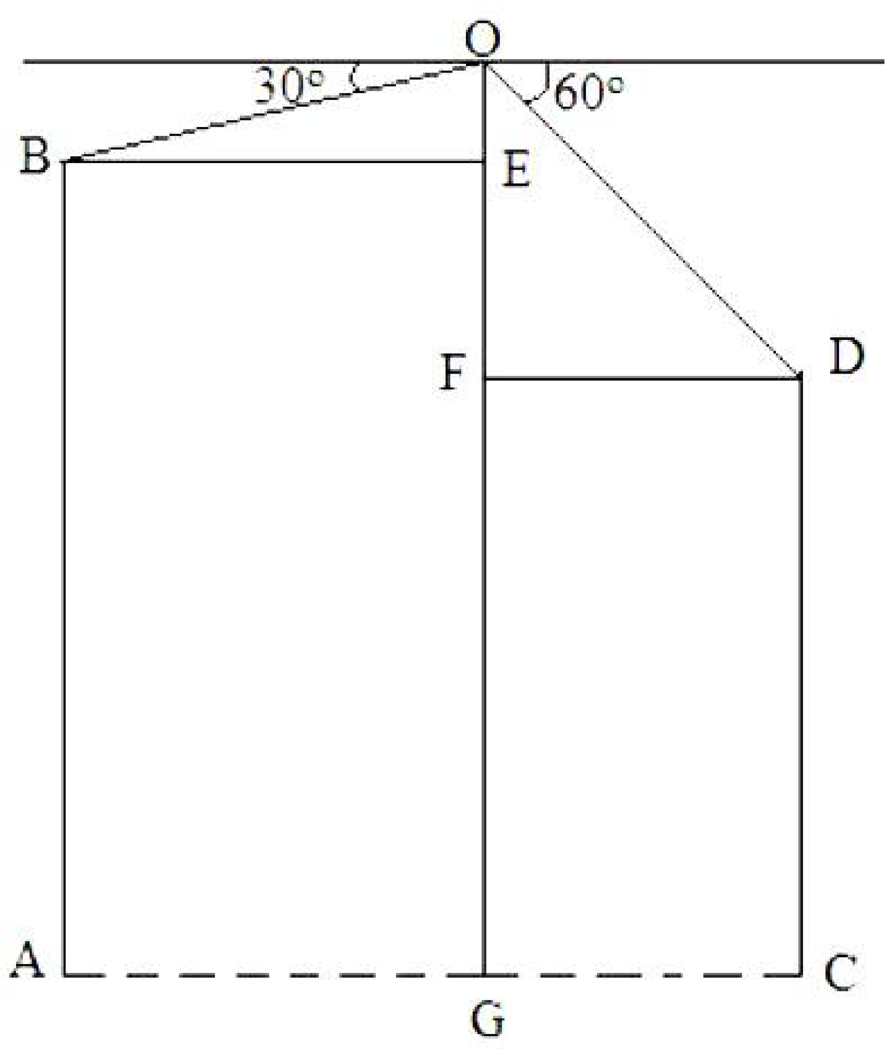
iii.

$C(4,5)$ and $B(4,1)$

$$d(BC) = \sqrt{(4-4)^2 + (5-1)^2} = 4 \text{ km}$$

38.

i. Let AB and CD represent buildings I and II respectively.



$$BE = \frac{AC}{2} = \frac{142}{2} = 71 \text{ m}$$

$$\cos(\angle OBE) = \frac{BE}{BO}$$

$$\Rightarrow \cos 30^\circ = \frac{71}{BO}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{71}{BO}$$

$$\Rightarrow BO = \frac{142}{\sqrt{3}} \text{ m}$$

OR

$$FD = \frac{AC}{2} = \frac{142}{2} = 71 \text{ m}$$

$$\cos(\angle ODF) = \frac{FD}{OD}$$

$$\Rightarrow \cos 60^\circ = \frac{71}{OD}$$

$$\Rightarrow \frac{1}{2} = \frac{71}{OD}$$

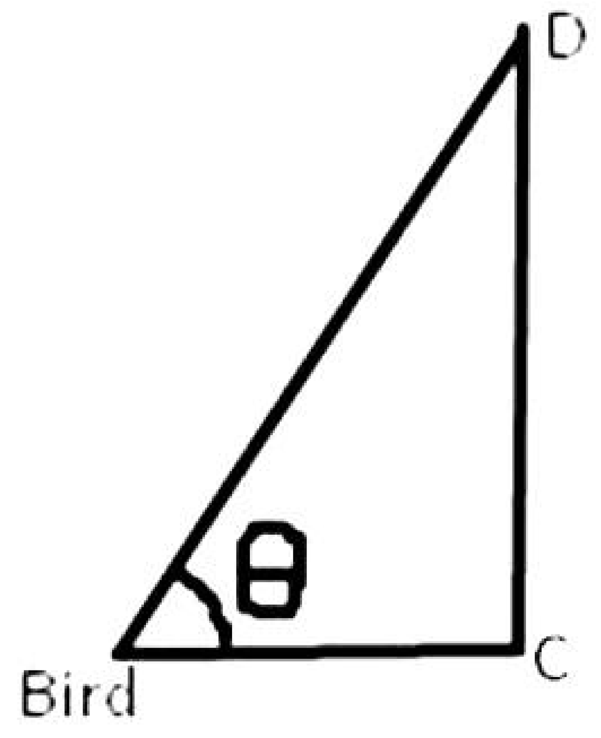
$$\Rightarrow OD = 142 \text{ m}$$

ii. Distance between bird and ground = OG = OE + EG

$$OE = BE \times \tan(\angle OBE) = 71 \times \frac{1}{\sqrt{3}} = \frac{71}{\sqrt{3}} = 40.99 \text{ m}$$

Therefore, OG = 534 + 40.99 = 574.99 m

iii.



$\tan \theta = 300/300 = 1$

Therefore, $\theta = 45^\circ$